

Continuous Time Stochastic Storage Processes With Random Linear Inputs and Outputs

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1. Introduction. The probabilistic treatment of various types of storage systems has received increased attention in the last decade through the impetus of problems in the theory of queues, dams, and inventory systems. Some of the more recent work has been summarized by J. Gani [5], but otherwise the material is scattered over a wide range of journals and books. Outstanding contributions to the theory of storage systems have been made by Arrow, Harris & Marschak [1], Arrow, Karlin & Scarf [2], D. G. Kendall [11], [13], P. A. P. Moran [14], [15], and L. Takács [17] to mention just a few.

The behavior of a storage process is determined by the input process and the output mechanism. Various models have been proposed to analyze a system in discrete or continuous time by imposing varied assumptions on the input and output, as for example gamma-type input and constant linear release by Moran [15] and Kendall [13]. Amidst the vast amounts of recent literature one rather simple model has been overlooked, and the purpose of this paper is to analyze it. Informally the model is described as follows: Let $X(t)$ be the level of the storage process at time t ; $X(0) = 0$. Starting at time 0 the level $X(t)$ increases linearly for a random length of time after which it decreases linearly for a random time. Subsequently, the process continues to alternate between periods of increase and decrease. All time intervals are independently distributed, and the periods of increase and decrease are identically distributed, respectively. Should $X(t)$ return to 0 at some time $t > 0$ it remains at this level until the end of the current period of decrease.

The fluctuations between increase and decrease can be interpreted in three ways. The first interpretation is that periods of input and output actually alternate and never overlap. Another is that there is always a constant linear output to the system provided the level is greater than zero but the input process shuts on and off. When there is input to the system, it is linear and exceeds the output so that the increase is actually the net increase to the system.

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