

# *On the Representation Problem for Stationary Stochastic Processes with Trivial Tail Field*

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**1. Introduction and summary.** Throughout this paper  $(\Omega, \Sigma, P)$  will be a probability space,  $\{X_n\}$  for  $n = 0, \pm 1, \dots$  will be a real valued strictly stationary stochastic process on  $(\Omega, \Sigma, P)$ ,  $\Sigma_n$  will be the sub- $\sigma$ -field of  $\Sigma$  generated by  $X_k$  for all  $k \leq n$ , and  $\Sigma_{-\infty}$  will be  $\bigcap \Sigma_n$ . Whenever  $\{X_n\}$  is a Markov process  $\bar{P}$  will be the stationary measure on Borel subsets of the real line defined by  $\bar{P}(A) = P\{X_n \in A\}$ ;  $\{P_x\}$  for  $x$  real will be the stationary transition probability measures on the real line defined by

$$P_x(A) = P\{X_{n+1} \in A \mid X_n = x\};$$

$\{P_x^{(n)}\}$  for  $x$  real will be the  $n$ -step stationary transition probabilities on the real line defined by

$$P_x^{(n)}(A) = P\{X_{k+n} \in A \mid X_k = x\}.$$

The process  $\{X_n\}$  will be called tail trivial and will be said to have a trivial tail field if  $A \in \Sigma_{-\infty}$  implies  $P(A) = 0$  or  $1$ .

Let  $\{\xi_n\}$  be an independent sequence of random variables uniformly distributed on  $[0, 1]$ , let  $\xi = (\dots, \xi_{-1}, \xi_0, \xi_1, \dots)$ , and define  $T$  by  $T\xi = (\dots, \xi_0, \xi_1, \xi_2, \dots)$ . When does there exist a function  $f$  on the sequence  $\{\xi_n\}$  such that the sequences  $\{X_n\}$  and  $\{f(T^n\xi)\}$  have the same probability structure (*i.e.* such that the joint distribution of  $X_{i_1}, \dots, X_{i_k}$  is the same as the joint distribution of  $f(T^{i_1}\xi), \dots, f(T^{i_k}\xi)$  for all  $k$  and all sequences  $i_1, \dots, i_k$ )? Rosenblatt has shown (see [2] and [3]) that if  $\{X_n\}$  is a stationary, ergodic, and aperiodic Markov chain with a denumerable state space, then  $\{X_n\}$  has a representation of the above type and in fact  $f(T^n\xi) = f(\dots, \xi_{n-1}, \xi_n)$ .

The following theorem and corollaries are proved in this paper. The theorem generalizes Rosenblatt's result to a type of Markov process with non-denumerable state space.

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