

On a Finite Lebedev Transform

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In [1] integral transforms of the Mellin type were developed to remove the terms $r^2\psi_{rr} + r\psi_r$ and similar terms occurring in certain partial differential equations. These transforms were adapted to the finite interval $(0, a)$ or the infinite interval (a, ∞) as well as to various boundary conditions. Here these Mellin type transforms are generalised to handle the group of terms $r^2\psi_{rr} + r\psi_r \pm k^2r^2\psi$ that occur in the space forms of the wave equation. The required generalisations are integral transforms of the Lebedev type. For some illustrations of the applications of this transform see [6, 7]. We confine attention to the finite interval $(0, a)$ this being the only case in which an integral theorem can be generated, other than for the range $(0, \infty)$. Also the boundary conditions considered are those for which either the function or its derivative is prescribed. The general method adopted is that employed in [1] by expressing the solution of a boundary value problem of the type of interest in terms of a suitable Green's function and modifying the form of the result by using an alternative integral representation of the Green's function. The Lebedev transform is closely associated with Watson's transformation [3] which converts one eigenfunction expansion into a more rapidly convergent one by contour integration. The essential step of the method employed here to construct the desired integral transform is to convert one form of a Green's function into another form by contour integration of the type used by Watson.

1. To generate an integral transform adapted to remove the operator

$$r^2\psi_{rr} + r\psi_r - k^2r^2\psi$$

in the finite interval $0 \leq r \leq a$ follow the method of [1] and consider the partial differential equation

$$(1) \quad r^2\psi_{rr} + r\psi_r - k^2r^2\psi + \psi_{\theta\theta} = 0$$

in the region $0 \leq r \leq a, 0 \leq \theta \leq \alpha$ with ψ determined by the conditions

$$\begin{aligned} \psi &= 0, & r &= a, & 0 &\leq \theta \leq \alpha, \\ \psi &= 0, & \theta &= \alpha, & 0 < r < a, \\ \psi &= f(r), & \theta &= 0, & 0 < r < a. \end{aligned}$$