

Positivity Intervals of Stable Processes

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1. Introduction. Let $\{X(t), t \geq 0\}$ be a separable symmetric stable process of index α ($0 < \alpha \leq 2$), *i.e.* a stochastic process with stationary independent increments whose distribution is determined by

$$E \exp \{i\xi(X(t) - X(s))\} = \exp \{(t - s) |\xi|^\alpha\} \quad s < t.$$

(*cf.* [4], pp. 96, 419). Unless otherwise mentioned we assume $X(0) = 0$ with probability one. Sample paths are denoted by $X(t, \omega)$. Consider the sets

$$A(\omega) = \{t : 0 \leq t \leq 1, X(t, \omega) > 0\},$$

i.e. the set in $[0, 1]$ where a particular sample path takes positive values. With probability one the limits

$$\lim_{t \uparrow s} X(t, \omega) \quad \text{and} \quad \lim_{t \downarrow s} X(t, \omega)$$

exist for all s ([4], p. 422). We shall assume that $X(t)$ is right continuous with probability one. Such a version of the process exists because there are no fixed discontinuities ([4], p. 422). There exists then for every $t \in A(\omega)$ an interval $I(t, \omega)$ of positive length such that

$$t \in I(t, \omega) \subseteq A(\omega).$$

(except if I contains the point 1) I is open at its right end point but at its left end point it may be open or closed. Thus $A(\omega) = \bigcup_{t \in A(\omega)} I(t, \omega)$ and by a well known argument ([5], pp. 103, 104) we can write A uniquely as a countable union

$$(1.1) \quad A(\omega) = \bigcup I_i(\omega)$$

of disjoint maximal intervals, *i.e.* there do not exist an i and an interval J such that $I_i(\omega) \subsetneq J \subseteq A(\omega)$.

It is well known (*e.g.* [7], p. 202) that the measure of A has an arcsin distribution. However, little seems to be known about the intervals $I_i(\omega)$. One can

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