

The Steady Magnetodynamic Flow of Perfectly Conducting Fluids

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1. Introduction. In the theory of inviscid, thermally non-conducting gas flows, the substitution principle [1] has shown that certain groups of flows possess the same streamline configurations. A consequence of this property is the development of a purely kinematic form of the equations of motion for the system. In this paper it is proposed, as far as seems possible, to extend these methods to the motion of a conducting gas subject to a magnetic field. Only the simplest system will be considered, in which the gas has infinite conductivity and no dissipative mechanisms are present. A substitution principle is established for a spatial flow but it takes the form of a surface property unlike the curvilinear one in gas dynamics. The velocity field can be changed by a constant multiple along certain surfaces without altering the streamlines, the pressure or magnetic fields.

By analogy with ordinary gas dynamics, the usefulness of this result depends upon the uniformity of the stagnation enthalpy along streamlines in order that we may select a transformation which can be used to reduce the number of dependent variables and equations. It appears that such flows are only possible when the magnetic field is either transverse or parallel to the direction of the velocity vector at a point. For a plane flow with a transverse magnetic field the elimination of the pressure can be achieved but only for particular forms of the equation of state which do not include the usual power-law form. There is one exceptional case when the adiabatic index is 2. Some of these results have been obtained previously by Vinokur [2] for isoenergetic flows.

More progress can be made with the class of problems in which the magnetic field is parallel to the velocity at every point. The pressure cannot be eliminated from the equations of motion except in a sub-class of problems in which the stagnation pressure is uniform and the magnetic field takes a particular form. A modified velocity vector constitutes a Beltrami field. The resulting equations have great similarity with the conventional equations of isentropic compressible fluid flow. The properties of these equations have received considerable at-