

# *Theory of Stability and Consolidation of a Porous Medium Under Initial Stress<sup>1</sup>*

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**Abstract.** Fundamental equations are derived for the mechanics of a fluid-filled porous medium under initial stress. The theory takes into account elastic and viscoelastic properties, including the most general case of anisotropy. It includes the theory of stability, and of acoustic propagation under initial stress, and by thermodynamic analogy the dynamics of a thermoelastic continuum under initial stress. Equations are also developed for a medium which is isotropic in finite strain. General variational principles are derived by which problems are easily formulated in curvilinear coordinates or in Lagrangian form by using generalized coordinates. It is shown that the variational principles are a direct consequence of the general equations of the thermodynamics of irreversible processes, and lead to real characteristic roots for instability.

**1. Introduction.** In the mechanics of porous media which has been developed to date the effect of the initial stress has not been introduced into the basic equations. The generalization of the theory to include this effect is of considerable interest in many applications. In civil engineering and geophysics the problems of consolidation and tectonics involve earth masses which are initially under high initial stress. In problems of foundation engineering the influence of the initial stress appears in a buoyancy effect which is actually used in design procedures and amounts to "floating" a building on its foundation. Since earth masses are generally porous and fluid-saturated, a consolidation theory taking into account this initial stress is obviously needed. On the other hand, problems of tectonic folding in geology are to a large extent problems of stability of porous media under initial stress. This requires an extension of the stability theory of continua to porous media. The theory presented in this paper is essentially the mechanics of fluid saturated porous media initially in equilibrium in a stressed condition and subjected to small perturbations. However, by a trivial limiting process where the variables become infinitesimals the theory leads also to instantaneous time rate equations valid for finite deformations.

The consolidation theory [1] [10] with suitable adaptations is combined with

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