

# *Binary Collision Modeling*

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**1. Introduction.** Variable density wind-tunnels have long been used to simulate flows past full-scale aircraft with small models. See, for example M. Munk & E. W. Miller, "The variable density wind-tunnel of the N.A.C.A.", NACA Repts. 227 and 228, Washington, 1925. By maintaining the wind-tunnel pressure at  $\beta$  atmospheres, one conserves both the Reynolds and Mach numbers with models whose linear scale is reduced in the ratio  $1 : \beta$ , without having to heat or cool the air.

A few years ago, it occurred to us that the same principle could be used to simulate upper atmospheric flight conditions very cheaply. One need only *reduce the linear scale in inverse proportion to the pressure*, while cooling or heating the air to agree with the normal temperature for the altitude to be simulated. For example, this would permit simulation of hypersonic flow around a 25" diameter missile at 100,000 feet altitude, by a  $\frac{1}{4}$ " diameter model fired at the same velocity under near-atmospheric conditions.

We recognized that the preceding scaling prescription conserved not only the Reynolds and Mach numbers of continuum mechanics, but also the averaged number of binary collisions per molecule. We therefore called it *binary collision modeling* ([1], p. 109), having in mind the following important corollary: binary collision modeling should conserve all thermodynamic and chemical kinetic properties which depend only on the statistical properties of *binary* molecular collisions.

In the intervening time, we have subjected these ideas to critical study, with special reference to the simulation of *hypersonic* flight conditions. The present paper contains our principal conclusions.

## A. SIMILARITY WITH FROZEN CHEMISTRY

**2. Perfect gas.** We first consider the validity of "binary collision modeling" in a (chemically "frozen") *thermodynamically perfect gas*, such as is postulated in most theoretical studies of supersonic flow. Physically the assumption of a chemically frozen perfect gas is fulfilled very well in free flight for missile speeds  $u < 7,000$  f/s (so that the stagnation temperature is less than  $2000^\circ\text{K}$ ), at