

Examples of the Behavior of Solutions of Hyperbolic Equations for Large Times¹

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We consider the problem of determining the behavior for large times of solutions of linear symmetric hyperbolic systems of differential equations with constant coefficients, defined in unbounded space. This behavior will be determined in terms of the singularities of a certain integral, which in turn depend on the geometry of the bicharacteristic cone of an associated symmetric hyperbolic system. We use the geometrical interpretation of our problem to produce examples of equations in 2 and 3 space dimensions for which the energy of solutions is independent of time, the volume of the support of solutions increases like t^2 and t^3 respectively, yet for which the fundamental solution decays like $t^{-1/2}$ at almost every point in space. It follows that in general, *i.e.*, except possibly for certain special solutions at special points, solutions decay like $t^{-1/2}$. This phenomenon may be thought of as a focussing of energy along the t -axis; it is related to conical refraction, which occurs in crystal optics and in other problems involving anisotropic media.

Knowledge of the behavior of solutions of hyperbolic equations for large times is important for the study of steady-state or time-harmonic problems, *e.g.*, radiation and scattering problems. This was pointed out to me by J. Keller. See, for example, C. H. Wilcox [6], p. 133 and R. M. Lewis [4], p. 623.

Our methods and terminology are taken from Courant-Hilbert [1]. The reader is referred to Chapter VI of that work for detailed explanations of our methods.

1. Definitions. We consider $n + 1$ real independent variables t, x^1, \dots, x^n (abbreviated to t, x), k complex dependent variables, denoted by the vector $u(t, x)$, and differential operators of the form

$$(1) \quad L = I \frac{\partial}{\partial t} + \sum_{r=1}^n A^r \frac{\partial}{\partial x^r} - iB_1 + B_2 .$$

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