

# *The Eigenvalue Problem for Sublinear Hammerstein Operators With Oscillation Kernels\**

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**1. Introduction.** In a previous paper the author derived some of the spectral properties of the following problem defined for  $0 \leq s \leq 1$ :

$$(1) \quad \begin{aligned} (p(s)u_s)_s + \lambda^{-1}f(s, u(s)) &= 0, \\ u(0) - ap(0)u_s(0) &= 0, \quad u(1) + bp(1)u_s(1) = 0, \end{aligned}$$

where  $a, b > 0$ ,  $p(s) > 0$  is differentiable, and  $f(s, u)$  is continuous in the strip  $0 \leq s \leq 1$ ,  $-\infty < u < +\infty$ . Moreover,  $f(s, u)$  was assumed to satisfy certain conditions designated H-1, H2, H-3 in that paper ([3], Section 1); these conditions classified the problem as "sublinear." It was mentioned that problem (1) is equivalent to a Hammerstein equation:

$$(2) \quad \lambda u(s) = \int_0^1 G(s, t)f(t, u(t)) dt,$$

where  $G(s, t)$  is the Green's function for the linear differential operator involved in (1).

The study of Hammerstein equations is important for at least two reasons: (a) these equations arise in physical problems, and (b) they serve as a first step into the realm of nonlinear functional equations, so that an understanding of them can be expected to shed light on properties of more complicated nonlinear problems.

The kernel  $G(s, t)$  in problem (2) is of a very special type. The purpose of this paper is to try to find out the extent to which our results for problem (2) continue to hold when we insert more general kernels  $K(s, t)$  in place of the very special kernel  $G(s, t)$ .

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