

*Some Results on the Asymptotic Behavior of Eigenvalues for a Class of Integral Equations with Translation Kernels**

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Introduction. The results obtained in this paper are an extension of those obtained in a previous paper [3]. The interest is in finite kernel translation integral equation eigenvalue problems, that is, in the integral equation

$$(1) \quad \int_{-T}^T r(t - \tau) \varphi(\tau) d\tau = \lambda \varphi(t).$$

Here $\varphi(t)$ is an eigenfunction and λ the associated eigenvalue. We shall cite the main result established in [3] as Theorem 1 below.

Theorem 1. *Let $r(t)$ be a positive definite even function on $[-2T, 2T]$ of the form*

$$(2) \quad r(t) = (-1)^k c |t|^{2k-1} + b(t), c > 0,$$

with $b(t)$ a function $2k - 1$ times continuously differentiable whose $(2k - 1)^{\text{th}}$ derivative is absolutely continuous ($k = 1, 2, \dots$). Then the j^{th} eigenvalue λ_j (indexed in descending magnitude) of integral equation (1) is asymptotically the same as

$$(3) \quad 2(2k - 1)! c \left(\frac{2T}{j\pi} \right)^{2k}$$

as $j \rightarrow \infty$.

The following definition will be convenient in stating the principal result of this paper. Consider a function $g(t)$ defined on $[-2T, 2T]$. Let β be a positive number that is not an integer. Assume that $g(x)$ is differentiable up to order $[\beta]$ ($[\beta]$ is the greatest integer less than β), that is, $g^{(i)}(x)$ is absolutely continuous

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