

Some Integral Transformation Formulae for the Stokes-Beltrami Equations

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Some integral representations for the Stokes-Beltrami equations are given which transform n dimensional velocity potentials and stream functions into their $(n + 1)$ dimensional equivalents, and vice versa. From this it is shown how the general Dirichlet problem for the stream function and velocity potential in three dimensions transform completely into corresponding boundary value problems in two dimensions. Various integral representations appropriate to coordinate systems of interest are presented to facilitate calculation for a specific problem. A similar procedure for solutions of the Helmholtz equation is also indicated.

Introduction. Payne [1] has applied the results of generalised axisymmetric potential theory, initiated by Weinstein [2], [3] to a class of boundary value problems in electro-statics and potential theory. From the point of view of application the method essentially consists in replacing an axially symmetric harmonic in three dimensions by an axially symmetric in a higher dimension (usually 5). This is achieved by means of a correspondence principle due to Weinstein and the method which amounts to solving the adjoint boundary value problem has been especially fruitful for problems in which the boundary is a spindle, lens or torus. More recently Williams [4] has reduced some three dimensional boundary value problems in potential theory for the lens to corresponding problems for Laplace's equation in two dimensions. The method employed here consists in selecting an appropriate integral representation which reduces the problem to the solution of an Abel integral equation. Makhovikov [5] has also reduced some potential problems for the finite circular cylinder to similar problems for Laplace's equation in two dimensions. In the present work some integral transformations are presented which relate n and $(n + 1)$ dimensional harmonics and vice versa. The particular case $n = 2$ shows how an axisymmetric problem in three dimensions for Laplace's equation or Stokes' stream function can be reduced to a two dimensional problem for Laplace's