

# *Power Dissipated in Linear Systems With Periodic Coefficients*

IGNACE I. KOLODNER<sup>1</sup>

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**1. Introduction.** Let  $z$  be any continuous real valued solution of the  $n \times n$  system of ordinary differential equations,

$$(1) \quad x' = A(t)x + \cos(\omega t - \alpha)f(t), \quad \omega \in [0, \infty), \quad \alpha \in [0, 2\pi).$$

We assume that the entries of the matrix  $A$  and of the column  $f$  are piecewise continuous, real valued, periodic functions of period  $T$  which are integrable over any finite interval. In what follows,  $J$ ,  $J^+$  and  $\bar{J}^+$  are respectively the sets of integers, of positive integers and of non-negative integers.

In this note we prove the following

**Theorem.** Let  $Y$  ( $Y(0) = I$ ) be the fundamental matrix for the reduced system of (1) and assume that:

1. all the eigenvalues of  $Z = Y(T)$  have absolute values less than 1.
2.  $Q$  is an  $n \times n$  symmetric 'selector' matrix of piecewise continuous, bounded, real valued, periodic functions with period  $T$ .

Then

$$(2) \quad P(\omega) = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t z^{tr}(\tau)Q(\tau)z(\tau)d\tau$$

exists, is independent of the choice of the particular solution  $z$  of (1), and the function  $\omega \rightarrow P(\omega)$  is continuous on  $[0, \infty) - S$  where  $S = \{k\pi/T, k \in \bar{J}^+\}$ . Specifically,  $P = P_1 + P_2$  where  $P_1$  is continuous and  $P_2 \neq 0$  only on  $S$ ; if  $A$ ,  $f$  and  $Q$  are constant, then  $P_2 = 0$  on  $(0, \infty)$  although  $P_2(0) \neq 0$  in general.

Formulae (7) and (8) for  $P_1$  and  $P_2$  respectively will be found in §3 below. In §5 we discuss some consequences of the theorem for linear vibrating systems with periodic coefficients. For basic facts used here concerning linear system of

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