*The Representation of a Class of Two State Stationary Processes in Terms of Independent Random Variables**

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Introduction. Consider a stationary random process $\{X_n\ ; n = 0, \pm 1, \ldots\}$. It has recently been of some interest to find conditions on $\{X_n\}$ such that one can construct a process $\{Y_n\}$ with the same probability structure as $\{X_n\}$ on the space of independent identically distributed random variables $\{\xi_n\}$ (say uniformly distributed on [0, 1]) by a Borel function $f(\xi_0\ ,\ \xi_{-1}\ ,\ \cdots) = f(\xi)(\xi = (\cdots\ ,\ \xi_{-1}\ ,\ \xi_0\ ,\ \xi_1\ ,\ \cdots))$ and its shifts, that is,

$$Y_n = f(T^n\xi), \qquad n = 0, \pm 1, \cdots . \tag{1}$$

Here f is to be a function of $\xi_0\ ,\ \xi_{-1}\ ,\ \cdots$ and T is the shift operator. It can be shown that a necessary condition is that $\{X_n\}$ be purely nondeterministic or regular (in the terminology of A. N. Kolmogorov [5]). In fact it has been shown that the necessary and sufficient condition for such a representation in the case of a countable state Markov chain is that it be regular [6]. Most of the sufficient conditions for such a representation in the case of a continuous state Markov process contain something reminiscent of a Doeblin condition [2]. We explicitly mention and state an interesting sufficient condition of this type obtained recently by Hanson:

(i) $\{X_n\}$ *is a stationary regular Markov process*

(ii) *There exist Borel sets* A, B *of the state space and a nonnegative measure* ϕ *on the state space such that* $\bar{P}(B) > 0$ ($\bar{P}$ *is the stationary measure induced by the process on the state space*), $\phi(A) > 0$ *and for all* $x \in B$ *and* $A' \subset A$ *the transition measure*

$$P(x, A') \geqq \phi(A'). \tag{2}$$

It is clear that it would be extremely interesting to investigate a class of processes which would not satisfy a Doeblin-like condition even when they are

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