

Approximation of Non-parametric Surfaces of Finite Area

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1. In answering a conjecture of mine, [G], in the affirmative, J. H. Michael, [M], proved the following:

If f is a measurable real function, which vanishes outside a compact set K , whose Lebesgue area $A(f; E)$ is finite on compact sets E , then if $\epsilon > 0$ and Q is a closed square such that K lies in its interior, there is a Lipschitzian g , with support in Q , such that

$$m[(x, y) : f(x, y) \neq g(x, y)] < \epsilon$$

and

$$| A(f; Q) - A(g; Q) | < \epsilon.$$

(For the definition of Lebesgue area for non-parametric surfaces given by measurable functions see [C] or [G].)

The purpose of this note is to show that the technique used by Michael may be slightly modified so that the approximating function g may be taken to be continuously differentiable, and that this is the best possible result. A discussion with Harry Pollard contributed to the present form of Theorem 2.

2. In this section, we recall some known facts about the class of functions under consideration here (for details, see [K] and [G]). Let \mathcal{G} be the set of functions, which vanish outside compact sets, for which $A(f; E) < \infty$ for every compact set E . Then,

(a) $f \in \mathcal{G}$ if and only if the partial derivatives of f , in the distribution sense, are totally finite measures μ_1 and μ_2 .

(b) If m is Lebesgue measure, then $A(f; E) = \alpha(E)$, for Borel sets E , where α is the total variation of the vector measure (μ_1, μ_2, m) .

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