Continuum Dynamics of Elastic Plates and Multilayered Solids Under Initial Stress*

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Abstract. General solutions are developed for the continuum dynamics of elastic plates and multilayered media under initial stress. By the use of specially suited analytical devices they are obtained in a remarkably simple form which brings out their mathematical structure and their physical significance. The results are derived for orthotropic materials and include the special case of a material isotropic in finite strain. They provide basic solutions, incorporating the effect of initial stress, in problems of vibrations and acoustics of sandwich plates and seismic propagation in stratified rock. The problem of elastic stability of plates and multilayered media is solved as a particular case by putting the frequency equal to zero. This extends to compressible media the analysis of internal, interfacial, and surface instability and other related phenomena developed earlier by the writer in the context of incompressibility. By viscoelastic correspondence the result also provides complete solutions for the dynamics of viscoelastic plates and multilayered viscoelastic media under initial stress.

1. Introduction. A continuum theory for the dynamics of elastic plates under initial stress is of considerable interest in many fields. It is essential in order to evaluate the influence of initial stresses on the acoustic properties of the plate, and should provide a rigorous foundation for problems of buckling and dynamic instability of plate structures. By extension to multilayered media constituted by a superposition of elastic layers the theory becomes applicable to problems of vibration and acoustics of sandwich structures and to seismic propagation in stratified sedimentary rock in the presence of initial stress.

It is our purpose here to derive general equations for the dynamics of elastic plates and multilayered continua under initial stress.

While the results are rigorous and very general the analytical expressions involved are remarkably simple in view of the complexity of the phenomena involved.

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