

*The Singular Cauchy Problem For a Quasi-Linear Hyperbolic Equation of Second Order**

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1. Introduction. The singular Cauchy problem for the linear hyperbolic equation

$$h(x, y)K(y)u_{xx} - u_{yy} + a(x, y)u_x + b(x, y)u_y + c(x, y)u + f(x, y) = 0,$$

($y > 0$), where h is positive and K is monotone increasing with $K(0) = 0$, was shown by Protter [1] to have a solution provided that

$$\lim_{y \rightarrow 0} \frac{ya(x, y)}{(K(y))^{\frac{1}{2}}} = 0.$$

The problem is singular in the sense that the initial values are prescribed on a segment of the x -axis where the equation is of parabolic type.

In this paper we consider the singular Cauchy problem for the quasi-linear equation

$$(1.1) \quad r^2(x, y)u^{2m}u_{xx} - u_{yy} + f(x, y, u, u_x, u_y) = 0,$$

($y > 0$), with the initial conditions

$$(1.2) \quad u(x, 0) = 0, \quad u_y(x, 0) = \varphi(x), \quad x \in I,$$

where $I = [a, b]$ is a finite closed interval. We assume that m is a positive constant, r is positive and r, f and φ are twice differentiable with Lipschitz continuous second derivatives on domains to be specified. In order to assure the hyperbolicity of (1.1) for $y > 0$, we also assume that there are constants α and β such that

$$0 < \alpha \leq \varphi(x) \leq \beta, \quad x \in I.$$

Choose B so that $B > \beta$ and let Γ_1 be the solution of the ordinary differential equation

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