

# *Uniqueness Theorems for the Helmholtz Equation in Domains With Infinite Boundaries\**

FAROUK M. ODEH

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**Introduction.** The role of Sommerfeld's radiation condition in problems of diffraction of time-harmonic waves is well known. When combined with appropriate boundary conditions, it ensures the uniqueness of the solution of the reduced wave equation in a domain exterior to a scattering object with a finite smooth boundary [10]. However, if the boundary extends to infinity, uniqueness has been proved only for a few special cases. Rellich [10] considered the case when Dirichlet data are given on a boundary which roughly resembles a paraboloid. Miranker [8] considered an exterior domain which contains a cone with an angle greater than  $\pi/2$ , but he had to restrict the values of the normal derivative of a solution on the boundary even when discussing a Dirichlet problem. Jones [2] treated conical domains. Levine [7] generalized Rellich's proof in [10] to cases when Dirichlet or Neumann boundary conditions are given on boundaries which eventually become "generalized cones".

In the present paper we consider the uniqueness question in three special cases which involve infinite boundaries and are not covered by the results cited above. First, we discuss Sommerfeld's "flat earth" problem [12]. We show that imposing the radiation condition, with two different wave numbers in two half-spaces consisting of different materials, does lead to uniqueness when the proper continuity conditions are imposed on the common boundary. Secondly, we prove the uniqueness of the solution to the Dirichlet problem for a domain which "opens up" at infinity. Lastly, we consider impedance type boundary conditions on the surface of an imperfectly conducting plane. These problems do not seem to be amenable to the method of eigenfunction expansions which is usually used in discussing uniqueness questions. Instead

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