

Some Functions Associated With the Bessel Functions¹

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Communicated by I. N. SNEDDON

1. **Introduction.** In the present paper we discuss some properties of the function

$$(1.1) \quad R(\lambda, \mu, \nu, x) = \sum_{n=0}^{\infty} \frac{(-1)^n (\lambda + n + 1)_n x^n}{n! \Gamma(\mu + n + 1) \Gamma(\nu + n + 1)}.$$

This function is suggested by the well known formula for the product $J_\mu(x)J_\nu(x)$; indeed we have ([8], p. 147)

$$(1.2) \quad J_\mu(2x)J_\nu(2x) = x^{\mu+\nu} R(\mu + \nu, \mu, \nu, x^2).$$

We note also that

$$(1.3) \quad R(2\mu, \mu, \mu - \frac{1}{2}, x^2) = \frac{x^{-2\mu}}{\pi^{\frac{1}{2}}} J_{2\mu}(4x)$$

and

$$(1.4) \quad R(2\mu, \mu, \mu + \frac{1}{2}, x) = \frac{2x^{-2\mu}}{\pi^{\frac{1}{2}}} \int_0^1 J_{2\mu}(4xt) dt.$$

Many of the results obtained in the paper are generalizations of known results concerning Bessel functions. For example the formula ([8], p. 151)

$$(\frac{1}{2}x)^{\mu+\nu} = \frac{\Gamma(\mu + 1)\Gamma(\nu + 1)}{\Gamma(\mu + \nu + 1)} \sum_{n=0}^{\infty} \frac{(\mu + \nu + 2n)\Gamma(\mu + \nu + n)}{n!} J_{\mu+n}(x)J_{\nu+n}(x)$$

admits of the generalization

$$(1.5) \quad \sum_{n=0}^{\infty} \frac{(\lambda + 2n)\Gamma(\lambda + n)}{n!} x^n R(\lambda + 2n, \mu + n, \nu + n, x) = \frac{\Gamma(\lambda + 1)}{\Gamma(\mu + 1)\Gamma(\nu + 1)},$$

which may be considered the starting point of the paper. By means of (1.5)

¹ Supported in part by NSF grant G16485.