

# Further Results on the Representation Problem For Stationary Stochastic Processes With Trivial Tail Field

J. R. BLUM<sup>1</sup> AND D. L. HANSON<sup>2</sup>

Communicated by M. ROSENBLATT

1. **Introduction.** Throughout this paper  $\{X_n\}$  for  $n = 0, \pm 1, \dots$  will be a strictly stationary stochastic process on a probability space  $(\Omega, \Sigma, P)$ , the  $\sigma$ -field generated by  $\{X_n\}$  will be  $\Sigma$ , the sub- $\sigma$ -fields of  $\Sigma$  generated by  $X_k$  for all  $k \geq n$  and all  $k \leq n$  will be  $\Sigma^n$  and  $\Sigma_n$  respectively,  $\Sigma_{-\infty} = \bigcap \Sigma_n$ , and  $T$  will be the shift transformation on  $\Sigma$  induced by  $\{X_n\}$  in such a way that  $\{X_0 \in B\} = T\{X_1 \in B\}$  etc. The process  $\{X_n\}$  is called tail trivial if  $\Sigma_{-\infty}$  contains only sets of measure zero and one.

Let  $\{\xi_n\}$  for  $n = 0, \pm 1, \dots$  be an independent sequence of random variables uniformly distributed on  $[0, 1]$ , let  $\xi = (\dots, \xi_{-1}, \xi_0, \xi_1, \dots)$ , and define  $T\xi = (\dots, \xi_0, \xi_1, \xi_2, \dots)$ . When does there exist a function  $f$  on the sequence  $\{\xi_n\}$  such that the sequences  $\{X_n\}$  and  $\{f(T^n\xi)\}$  have the same probability structure (i.e. such that the joint distribution of  $X_{i_1}, \dots, X_{i_k}$  is the same as the joint distribution of  $f(T^{i_1}\xi), \dots, f(T^{i_k}\xi)$  for all  $k$  and all sequences  $i_1, \dots, i_k$ )? Rosenblatt has shown (see [3]) that if  $\{X_n\}$  is a stationary, ergodic, and aperiodic Markov chain with a denumerable state space then  $\{X_n\}$  has a representation of the above type and in fact he obtains an  $f$  of the form  $f(T^n\xi) = f(\dots, \xi_{n-1}, \xi_n)$ . In [1] Rosenblatt's result is generalized to a class of tail trivial Markov processes which includes ergodic, aperiodic Markov chains and which also includes some processes with a nondenumerable state space.

Our interest lies mainly in representations of the form obtained by Rosenblatt (i.e. of the form  $f(T^n\xi) = f(\dots, \xi_{n-1}, \xi_n)$ ) which can be thought of as representations in terms of the past and present  $\xi_k$ 's. It is easy to show (see for example Lemma 1 of [2]) that in order to obtain a representation of the desired type the original process  $\{X_n\}$  must be tail trivial.

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<sup>1</sup> Research supported by the National Science Foundation, Grant NSF G-21219.

<sup>2</sup> Sandia Corporation. Research done under the auspices of the United States Atomic Energy Commission.