

*A Central Limit Theorem for Non-linear Functions of a Normal Stationary Process**

TZE-CHIEN SUN

Communicated by M. ROSENBLATT

Introduction. Let $X(t)$, $t = 0, \pm 1, \pm 2, \dots$ be a real normal stationary process with mean $EX(t) \equiv 0$ for all t and covariances

$$r(k) = \text{cov}(X(j), X(j+k)) = \frac{1}{2} \int_{-\pi}^{\pi} e^{ik\lambda} f(\lambda) d\lambda, \quad k = 0, \pm 1, \pm 2, \dots$$

where $f \in L^2(-\pi, \pi)$. Since $X(t)$ is a real process, $f(\lambda)$ is symmetrical with respect to $\lambda = 0$, *i.e.* $f(-\lambda) = f(\lambda)$ for $-\pi \leq \lambda \leq \pi$. Since $r(k)$ is a positive definite sequence, $f(\lambda) \geq 0$ for $-\pi \leq \lambda \leq \pi$.

Let

$$(1) \quad c_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\lambda) d\lambda \leq C_1 = \max(1, c_1)$$
$$c_2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^2(\lambda) d\lambda \leq C_2 = \max(1, c_2).$$

Note that $f(\lambda)$ above is defined only on $[-\pi, \pi]$. However we may at times find it necessary for later use to extend $f(\lambda)$ to the whole real line by periodicity with period 2π , *i.e.*

$$f(\lambda + 2n\pi) = f(\lambda), \quad -\pi \leq \lambda < \pi, \quad n = 0, \pm 1, \pm 2, \dots$$

It has been known that a stationary process generated by any linear function of a normal stationary process is again normal if it exists in the mean. But a stationary process generated by a non-linear function of a normal stationary process is not in general normal, *e.g.* $X^2(t)$ has as its marginal distribution a chi-square distribution and hence is not normal. However, it is a very interesting question to ask what conditions we should impose on the original

* This research was supported by the Office of Naval Research under Contract Nonr 562(29) at Brown University. Reproduction in whole or in part is permitted for any purpose of the United States Government.