

# *A Spherical Inclusion Problem in Potential Theory*

E. E. JONES

*Communicated By* I. N. SNEDDON

**1. Introduction.** There exists a class of physical problems which can be expressed in the form of boundary-value problems involving regions internal and external to a spherical surface. Included in this class are problems from the fields of gravitation, magnetostatics, electrostatics, current and fluid flow, and heat conduction, the controlling partial differential equations being either of the Laplace or the Poisson type. Several authors have obtained solutions to the Laplace equation in terms of prescribed functions defined in one or both of the regions internal and external to a sphere, which together with certain partial derivatives satisfy given conditions on the spherical surface. Solutions to a problem of some generality, which included all previous solutions as special cases, were given by Yeh, Martinek & Ludford [1], and solutions to the corresponding problem when the controlling equation was of the Poisson type were given by Jones [2]. A feature of the analyses in [1], [2], is that no consistent method is presented for the setting up of the final solutions, rather, the solutions are built up from elementary solutions of the controlling differential equations, and are adjusted to satisfy the conditions of the problem.

Generally this method is difficult to apply, as there is not always present an indication of the type of solution required. Some progress towards a consistent method has been achieved by Churchill [3], by the use of Legendre transform calculus. However solutions were obtained only for axially-symmetrical problems, and those for non-symmetrical problems were proposed by generalizing the forms of the former solutions. The object of the present paper is to present a method of obtaining solutions to a boundary-value problem for a sphere, the boundary conditions on the sphere involving linear combinations with constant coefficients of these solutions and their first order radial derivatives.

It is assumed that polar coordinates  $(r, \theta, \omega)$  map out the whole of space with pole at the center of a sphere  $S$  of radius  $a$ . It is required to determine functions  $\phi_1(r)$ ,  $\phi_2(r)$  defined in regions  $V_1$ ,  $V_2$ , respectively internal and external to  $S$ , which satisfy the Poisson partial differential equations

$$(1.1) \quad \nabla^2 \phi_j(r) = f_j(r) \text{ in } V_j, \quad (j = 1, 2),$$