

Singularity-Free Regions For Solutions of 2nd Order Non-Linear Differential Equations¹

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1. Introduction. This paper is concerned with solutions of the complex equation

$$(1.1) \quad \frac{d^2 y}{dz^2} + F(y, z) = 0.$$

$F(y, z)$ is an entire function of y and regular-analytic in z for $|z| < R$. In addition, we assume that

$$(1.2) \quad |F(y, z)| \leq G(|y|, |z|).$$

for all $y, |z| < R$, where the majorant $G(s, t)$ is a continuous and non-decreasing function of s for $0 \leq s < \infty$ and continuous in t for $0 \leq t < R$.

A region is said to be *singularity-free* for a solution if the solution is regular-analytic in every closed subdomain. We seek solutions satisfying given initial conditions which are analytic in a certain simply-connected region. The basic result is Theorem 2.1 which compares the growth of the absolute values of these solutions with the growth of real-valued continuous functions verifying an associated differential inequality.

In particular, for the complex equation

$$(1.3) \quad \frac{d^2 y}{dz^2} + p(z)y^n = 0, \quad n > 1,$$

it is found that there exist solutions which satisfy given initial conditions and which are regular in the unit disk (Theorem 2.2).

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