

On the Geometry of the Steady, Complex-Lamellar Gas Flows

E. R. SURYANARAYAN

Communicated by G. BIRKHOFF

Introduction. In this paper we discuss the geometric properties of three dimensional steady complex-lamellar gas flows. We assume that the fluid is non-viscous and non-heat-conducting and subject to no extraneous force. Complex-lamellar flows are characterized by the fact that there exists a family of ∞^1 surfaces orthogonal to the stream lines [1]. Each surface is a Beltrami surface.

By expressing the normals to the surfaces of constant pressure and the normals to the Bernoulli surfaces in terms of the tangent, the principal normal and the binormal vectors of the stream lines, it is found that the Bernoulli surfaces are orthogonal to the surfaces of constant pressure if the stream lines are straight lines. Moreover, in the case of isentropic flows, the Bernoulli surfaces are orthogonal to the surfaces of constant pressure, if and only if the stream lines are straight lines, or, the curvature of the stream lines is equal to the directional derivative of $\ln q$ along the principal normals to the stream lines, q being the magnitude of the velocity vector. In isentropic complex-lamellar gas flows, it is found that the Beltrami surfaces and the Bernoulli surfaces cut at right angles along the vortex lines. The mean curvature and the Gaussian curvature of the Beltrami surfaces are determined in terms of q , the velocity vector, the density and the pressure.

The intrinsic equations of motion and continuity are obtained in the case when the Beltrami surfaces form a family of surfaces of revolution, the family of meridian curves being hyperbolas. From these equations, two special flows are obtained: one, the isentropic flow of a gas, and the other, the fluid flow in the incompressible case.

2. The Equations. Let x^j ($j = 1, 2, 3$) denote the variables of a system of Cartesian orthogonal coordinates. In order to use the summation convention, we shall write indices in covariant and contravariant positions. The velocity vector will be denoted by u^i . Let us write

$$(2.1) \quad u^i = qs^i,$$

where s^i is the unit vector in the direction of u^i , and q is the magnitude of the velocity vector.