Local Behavior of Plane Gravity Flows II. Infinite Jets on Fixed Boundaries*

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1. Introduction. Consider a steady plane irrotational flow of an incompressible inviscid fluid under gravity, where the flow boundary contains a "free boundary arc" Γ_{σ} and an analytic "fixed boundary arc" Γ_{σ} . A previous communication [1] dealt with the local flow behavior near a finite common endpoint of Γ_{σ} and Γ_{σ} , where the flow velocity is continuous and nonvanishing. The present paper treats the case in which Γ_{σ} and Γ_{σ} meet at infinity, and form the boundaries of a jet of fluid having a source or sink at infinity, where the flow velocity becomes infinite.

Specifically, we assume that the fixed boundary is analytic at infinity, but make no explicit assumption regarding its direction at infinity relative to the direction of gravity or to the free boundary. Instead of requiring a priori that the velocity must approach infinity throughout the jet, we assume this only on the free boundary; elsewhere in the jet the flow is assumed merely to be free of singularities and stagnation points. Among our principal results is the fact that the velocity does indeed approach infinity throughout the jet, that the jet becomes thin and asymptotic to the free boundary, and that the whole configuration extends to infinity with a positive component in the direction of gravity.

Following the notation of [1], we take a complex coordinate z in the flow plane, chosen in this case so that the fixed boundary is asymptotic at infinity to the positive real axis. The flow near infinity is determined by the inverse velocity potential function z=f(w), which maps a closed semi-infinite horizontal strip S in the w-plane conformally onto the jet so that the horizontal boundaries of S map individually into Γ_a and Γ_f , and $f(\infty)=\infty$. The half-lines in the w-plane which correspond in this way to Γ_a and Γ_f are denoted by L_a and L_f , respectively. The half-strip extends to Re $\{w\}=-\infty$ or $+\infty$

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