

On a Lebedev Expansion Theorem

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In [4] some extensions of the Kontorovich-Lebedev transform were developed to eliminate the r -dependence that occurs when the space form of the wave equation is written in a polar coordinate system. The transforms were applicable to finite regions bounded by the natural coordinate surfaces of a cylindrical or spherical coordinate system. Here we consider related transforms adapted to deal with infinite regions bounded internally for which the range of variation is $a < r < \infty$. We confine attention to boundary conditions for which either the function or its derivative is prescribed on the boundary $r = a$. The expansion theorem constructed is incorporated in the equations (18) and (19). The general method adopted is to replace the eigenfunction expansion by a contour integral and then follow the procedure developed in [4]. In any specific case the eigenfunction expansion of the type (19) can be obtained from corresponding classical type expansions by applying the transformation of Watson [1]. The integral transforms developed here enable the Lebedev type solution to be obtained without recourse to the classical solution or Watson's transformation. In addition the inversion formula (19) is restated in the form of a contour integral, equation (23), from which either form of eigenfunction expansion can be obtained as required.

1. We commence by constructing the eigenfunctions and Green's function bounded in the interval $0 < a < r < \infty$ of the differential equation

$$(1) \quad r^2 w_{rr} + r w_r - (k^2 r^2 + s^2) w = 0,$$

subject to the condition of vanishing on $r = a$. Let us define an integral transform of a function $\psi(r)$ by means of the equation

$$(2) \quad \Psi(u) = \int_a^\infty K_u(kr) \psi(r) \frac{dr}{r};$$

then we can readily establish the formula

$$(3) \quad \int_a^\infty (r^2 \psi_{rr} + r \psi_r - k^2 r^2 \psi) K_u(kr) \frac{dr}{r} = u^2 \Psi(u) + ka \psi(a) K'_u(ka) \\ - a \psi'(a) K_u(ka) + \lim_{r \rightarrow \infty} [r \psi'(r) K_u(kr) - kr \psi(r) K'_u(kr)].$$