

# On Singular Integral Operators

MARVIN SHINBROT<sup>1</sup>

*Communicated by* M. ROSENBLATT

1. The canonical example of a singular integral operator is the operator defined by

$$(1.1) \quad (Lf)(x) = A(x)f(x) + B(x)(Hf)(x)$$

on  $L^2$ ; here,  $A(x)$  and  $B(x)$  denote bounded functions and

$$(Hf)(x) = \frac{1}{i\pi} \int_{-\infty}^{\infty} \frac{f(y)}{y - x} dy,$$

the integral being a Cauchy principal value [1]. One is particularly interested in knowing when the equation

$$(1.2) \quad Lf = g$$

has a unique solution and, if it does, in methods for finding it.

Many generalizations of this problem have been considered; in particular, generalizations to  $L^p$  [2], to functions of several variables [3], and to several simultaneous equations [4] can be found in the literature. In this paper, we remain entirely in Hilbert space and generalize (1.1) by placing it in an abstract setting and then deriving a constructive general method for solving (1.2) in this abstract form. Abstract versions of (1.1) have been studied before [5], but it would appear that no one has as yet attempted to find explicit solutions in this form.

To see how the abstraction comes about, notice that multiplication by the functions  $A(x)$  and  $B(x)$  can be considered as bounded normal operators on  $L^2$ . Moreover, these operators obviously commute. Also, as is well known (see [6] or §6 below),  $H$  can be written as the difference of two orthogonal *projections*,  $H_+$  and  $H_-$ , whose sum is the identity. Thus, (1.1) is a special case of an operator of the form

$$(1.3) \quad \begin{aligned} L &= A(H_+ + H_-) + B(H_+ - H_-) \\ &= (A - B)H_+ + (A + B)H_- \end{aligned}$$

---

<sup>1</sup> The work reported here was supported by the Office of Naval Research (ONR 222(60)).