

# *An Ergodic Theorem with Weighted Averages*

GLEN BAXTER<sup>1</sup>

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**1. Introduction.** In this note we will prove a point-wise convergence theorem analogous to the ergodic theorem, the novelty being that we use a system of weighted averages. The work of Chacon-Ornstein [2], Dunford-Schwartz [4], and Hopf [7] suggest that a more general approach could be adopted than the one we will use. However, so as not to obscure the basic ideas, we consider the simplest possible case, where  $(S, \mathfrak{F}, m)$  is a finite measure space,  $f(s)$  is an integrable function with respect to  $m$ , and  $T$  is a one-one measure preserving point transformation from  $S$  into itself.

Let  $\{f_k, 1 \leq k < \infty\}$  be a sequence of non-negative numbers for which

$$(1) \quad \sum_{k=1}^{\infty} f_k = 1,$$

and define a sequence  $\{u_k, 0 \leq k < \infty\}$  by

$$(2) \quad u_n = f_1 u_{n-1} + f_2 u_{n-2} + \cdots + f_n u_0, \quad u_0 = 1.$$

For every  $n, 0 \leq u_n \leq 1$ , and it is well-known that

$$(3) \quad \lim_{n \rightarrow \infty} u_n = \left( \sum_1^{\infty} k f_k \right)^{-1},$$

where the term on the right in (3) is zero if  $\sum k f_k$  is not convergent. We are interested in the behavior of the ratios

$$(4) \quad R_n(s) = \sum_{m=0}^n u_m f(T^m s) / \sum_{m=0}^n u_m,$$

as  $n$  tends to infinity.

In the main theorem, it is shown that the convergence almost everywhere of  $R_n(s)$  is assured if  $R_n(s)$  and  $R_n(Ts)$  have the same limiting behavior, that is, if

$$(5) \quad R_n(s) - R_n(Ts) \rightarrow 0 \quad \text{a.e.}$$

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