

On the Classification of Hermitian Symmetric Spaces

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1. Introduction. The hermitian symmetric spaces were first studied by É. Cartan [3], who classified them by means of his classification [2] of the Riemannian symmetric spaces. The work of Borel and de Siebenthal on subgroups of maximal rank in compact Lie groups [1] gives a simpler proof of Cartan's classification result. Other proofs result from H. C. Wang's work on C -spaces [7] and J. Tits' work on parabolic groups [6]. We will give a direct proof which appears to be the simplest available.

2. Preliminaries. In order to establish notation and terminology and to reduce our problem to a problem on compact simple Lie groups, we recall some basic notions on symmetric spaces. The reader is referred to Helgason's book [4] for details.

2.1. A *Riemannian symmetric space* is a connected Riemannian manifold M such that, given $x \in M$, there is a (globally defined) isometry s_x which preserves x and has differential $-I$ on the tangent space M_x ; s_x is the *symmetry to M at x* . Given a complex manifold with hermitian metric, one obtains a Riemannian manifold by taking the underlying real manifold and the real part of the hermitian metric. If this Riemannian manifold is symmetric, and if the symmetries are hermitian isometries, one says that the original complex manifold with hermitian metric is a *hermitian symmetric space*. A hermitian symmetric space is always Kählerian.

Let \tilde{M} be a complete simply connected Riemannian manifold. Then [5] \tilde{M} is isometric to a product $M_0 \times M_1 \times \cdots \times M_r$, where M_0 is a Euclidean space and the other M_i are irreducible (not Euclidean, and not locally isometric to a product of lower dimensional Riemannian manifolds.) \tilde{M} is symmetric if and only if each of the M_i is symmetric. \tilde{M} is the real structure of a Kähler manifold if and only if each of the M_i is the real structure of a Kähler manifold,

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