

On the Dual Pontrjagin Classes of a Manifold

HOWARD M. ROBERTS

Communicated by A. H. WALLACE

Introduction. This paper is devoted to proving that for manifolds of dimension n certain polynomials in the dual Pontrjagin classes must be zero modulo q an odd prime. The methods are similar to those used by William Massey in his paper "On the Stiefel-Whitney Classes of a Manifold" [7] in which he showed that for certain values of n and k the $(n - k)$ -dimensional dual Stiefel-Whitney class of an n -dimensional manifold is always zero. Later William Massey and Franklin Peterson showed that these results were, for non-orientable manifolds, the best possible [8]. In 1962 Edgar Brown Jr. and Franklin Peterson determined all the relations that the Stiefel-Whitney classes of a manifold must satisfy [3].

The results of this paper are a step toward solving the much more difficult problem of determining all the relations that the Pontrjagin classes of an n -dimensional manifold must satisfy.

Theorem 3 of this paper is slightly stronger than a similar theorem proven for the author's Ph.D. thesis submitted to Brown University [10]. Theorem 1 and 2 were suggested to the author by a paper of Yasurô Tomanaga who was working on the problem independently in the modulo 3 case only [11]. However, Theorem 1 taken modulo 3 is a little stronger than Tomonaga's.

1. M^n is a compact, connected, oriented, and differentiable manifold throughout the paper. Following the notation of John Milnor [9], let ξ be a real vector bundle with base space M^n , total space E_ξ , projection π_ξ and fibre R^m . Let $E_{\xi,0}$ denote the set of non zero vectors of E_ξ . There exists a canonical class

$$(1.1) \quad U_\xi \in H^n(E_\xi, E_{\xi,0}; Z_q),$$

where Z_q is the ring of integers modulo an odd prime q . The Thom isomorphism ϕ_ξ [9] is defined as follows:

$$(1.2) \quad \begin{aligned} \phi_\xi : H^*(M^n; Z_q) &\rightarrow H^*(E_\xi, E_{\xi,0}; Z_q), \\ \phi_\xi(x) &= [\pi_\xi^*(x)]U_\xi. \end{aligned}$$