

*Composition Formulas in Generalized Axially Symmetric Potential Theory**

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I. Introduction. In numerous papers Professor A. Weinstein [1] [2] [3] [4] [5] [6] has investigated the solutions of the partial differential equation,

$$(1.1) \quad L[u] \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{k}{y} \frac{\partial u}{\partial y} = 0, \quad k > 0 \text{ (GASPT)}$$

which he refers to as *Generalized Axially Symmetric Potential Theory*. In several recent papers R. Gilbert [7] [8] [9] has obtained some new properties of the functions of GASPT by transplanting properties of analytic functions through the use of integral operators. The method of integral operators he uses is essentially that developed by Professor S. Bergman [10] [11] [12] [13] by which analytic functions of one or several complex variables are transformed into solutions of partial differential equations. In particular, this author [7] [8] [9] has introduced the integral operator

$$u(z) \equiv u(x, y) = \mathcal{A}_k[f(\sigma)], \quad \mathcal{A}_k[f] \equiv \alpha_k \int_{\mathcal{E}} f(\sigma) [\zeta - \zeta^{-1}]^{k-1} \frac{d\zeta}{\zeta},$$

$$\alpha_k = \frac{4}{(4i)^k \Gamma(k/2)^2}, \quad \sigma = x + \frac{iy}{2} [\zeta + \zeta^{-1}],$$

$$(1.2) \quad \mathcal{E} \equiv \{\zeta \mid \zeta = e^{i\varphi}, 0 \leq \varphi \leq \pi\}, \quad |z - z^0| < \epsilon, \quad z = x + iy,$$

z^0 is an initial point of definition, and $\epsilon > 0$ is sufficiently small, which transforms analytic functions of one complex variable, $f(\sigma)$, into solutions of the GASPT equation, $u(z) \equiv u(x, y)$.

From our knowledge of special functions it is clear that if $f(\sigma)$ is regular about the origin,

$$(1.3) \quad f(\sigma) = \sum_{n=0}^{\infty} a_n \sigma^n,$$

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