

# *Brownian Transition Functions and Random Walks on Certain Homogeneous Spaces*

RAMESH GANGOLLI<sup>1</sup>

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**Introduction.** A Markov transition function  $P(t, m, B)$  on a homogeneous space  $G/K$  of a Lie group  $G$  modulo a compact subgroup  $K$  is said to be Brownian if it is invariant under the left action of  $G$  on  $G/K$ , and is of local character. (Definitions below).

We study such transition functions when  $G$  is a compact connected Lie group and  $K$  is a compact subgroup. As is well-known, any compact Riemannian manifold whose group of isometries is transitive can be subsumed as  $G/K$  in the above set-up.

Yosida [9] has studied Brownian transition functions. Roughly speaking, he showed that they could be described as “fundamental solutions” of diffusion equations involving elliptic second-order differential operators on  $G/K$  which commute with the left action of  $G$  on  $G/K$ . It is thus clear that in general there may exist several Brownian transition functions on  $G/K$ . In this paper we show that there is a connection between the size of  $K$  and the number of Brownian transition functions. More precisely, let  $Q$  be the space of orbits under the adjoint representation  $\text{Ad}(K)$  of points on the unit sphere in the tangent space at  $\pi(e)$  in  $G/K$  ( $\pi$  being the natural projection  $G \rightarrow G/K$  and  $e$  the identity in  $G$ ). Then to every probability measure  $\mu$  on  $Q$  corresponds a Brownian transition function  $P_\mu$  and conversely. Moreover, with the help of a given  $\mu$ , we construct a sequence of “random walks” on  $G/K$  which converge in distribution to the Brownian motion whose transition function is  $P_\mu$ . The construction is effected by means of explicitly deriving the form of the semi-group generated by  $P_\mu$  and as an incidental fact this semi-group is shown to leave invariant the space of continuous functions on  $G/K$ . Finally, given a Brownian transition function on  $G/K$  it is shown how one can recover a measure  $\mu$  on the orbit space  $Q$  which will serve for the above construction.

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