

*A Necessary Criterion for the Existence of Certain Minimal Surfaces*¹

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1. If one dips a frame of thin wire, consisting of one or several contours, into a specially prepared soap solution and pulls it out again, one obtains an exquisitely looking iridescent curved soap film spanned into the frame. This film very nearly is the physical realization of a surface which differential geometers call minimal surface. The name suggests that a minimal surface should minimize the area among all surfaces with the same boundary; and in many cases, although not always, it actually does.

As anybody who has ever tried his hand at soap film experiments knows, however, it is not every time that a soap film develops in the frame. There can be two different reasons for this failure: either a lack of skill on the part of the experimenter, or, more deeply seated, the mathematical fact that no minimal surface bounded by the curves of the frame exists at all.

This latter situation is clearly illustrated with the well known classical experiment, in which the frame consists of two coaxial unit circles Γ_1 and Γ_2 in parallel planes, say the planes $z = -h$ and $z = h$ (see for instance the presentation in G. A. Bliss [1]). For sufficiently small values of h there exists a minimal surface S_h of the type of the circular annulus bounded by the curves Γ_1 and Γ_2 , namely a catenoid. Its area $I(S_h)$ is an absolute minimum compared with that of all surfaces of revolution bounded by Γ_1 and Γ_2 , including the Goldschmidt solution of total area 2π . We now increase the distance $2h$ between the curves Γ_1 and Γ_2 . For a certain value $h = h' = 0.5277 \dots$ $I(S_h)$ becomes equal to 2π . If h is slightly larger than h' then S_h does no longer furnish the absolute minimum of the area but still a strong relative minimum. Eventually we come to a limit value $h = h'' = 0.6627 \dots$. For $h > h''$ there exists no doubly-connected minimal surface bounded by the curves Γ_1 and Γ_2 . All this can be discussed by

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