

# *Generalized Varieties as Limits*

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**1. Introduction.** Variational theory in the higher dimensions makes it necessary to substitute, for the classical concepts of parametric surface, or parametric variety, more convenient ones, which possess satisfactory compactness properties, and are not tied down by any restriction of finite topological type. The substitution can be effected in two, fundamentally distinct, ways, as a result of which the theory splits in two parts, which deal with what we have termed in [6] problems *A* and *B*. The classical method of fields, and the algorithm, studied in [6], on which, according to Carathéodory [1], this method rests, apply in the higher dimensions only to problem *A*. In that case we deal with a generalization of the notion of polytope with positive real coefficients, or as we term it, that of weighted polytope. The corresponding generalization of the notion of polytope with positive integer coefficients, or simply, of polytope, leads to a completely different problem, namely problem *B*.

In the simplest case, that of the least area problem for oriented varieties, problem *B* is equivalent to the one formulated by Federer and Fleming [2] in terms of integral currents. It has been shown in [9] that this problem is certainly quite distinct from problem *A*, so that it has solutions which cannot be obtained by the method of fields.

The admissible "generalized varieties" which correspond to the two problems, or, as we shall term them, the generalized varieties admissible *A*, and those admissible *B*, have a very different structure; and moreover a deep and difficult question remains open in the *A* case: this is formulated in [6] and in section 2 below.

It is remarkable that, in spite of these circumstances, we are able here to characterize the two classes of admissible generalized varieties as two highly similar closures of the corresponding classes of simplicial varieties. This is, indeed, the main object of the present paper, and the gist of the following pair of theorems:

**(1.1) Theorem A.** *In order that  $L$  be a generalized variety admissible  $A$ , it is necessary and sufficient that there exist, for  $\nu = 1, 2, \dots$ , a weighted polytope  $L_\nu$ ,*