A Sequence of Submerged Dipole Distributions Whose Wave Resistance Tends to Zero¹

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Abstract. We show that the problem of finding a dipole distribution (on a submerged horizontal line segment parallel to the incident flow) whose (linearized) volume is fixed and whose wave resistance is a minimum has no solution. We exhibit a sequence of such dipole distributions whose wave resistance tends to zero (see eq. (34).)

1. Introduction and summary. We consider the problem of minimizing the wave resistance of a submerged uniformly-translating truncated horizontal line dipole distribution which is absolutely integrable and has fixed positive integrated dipole strength (linearized volume). The dipoles have axes parallel to the horizontal line segment and to the direction of uniform translation. This problem arises in the theory of the wave resistance of bodies (ships and submarines), and for this reason it is natural to interpret our results in terms of slender-body theory. We keep this interpretation in mind, without committing ourselves to it, by referring to (a non-zero multiple of) the integrated dipole strength as the linearized volume. It is also possible in principle to precisely compute the streamlines, and closed bodies (if any), defined by a submerged dipole distribution, from the corresponding potential satisfying the linearized free surface condition, but no such calculations have been published. Approximate bodies, obtained by finding the streamlines of singularity distributions whose potentials satisfy the zero-Froude-number boundary condition $\partial \phi / \partial y = 0$, have been found by a number of authors, and in many practical cases the bodies differ considerably from those defined by thin-ship or slender body formulas. The use of the full finite-Froude-number potential will introduce additional perturbations in the dipole-body relationship.

In the discussion following [1] (see page 112 thereof) Weinblum stated that the minimum problem (proposed above) was being treated numerically at his laboratory. Kotik then replied (see page 117 of [1]) that the minimum

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