

On the Uniqueness of Solutions of A Non-Linear Boundary Value Problem

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1. This note concerns boundary value problems of the form

$$(1.1) \quad v'' + f(x, v) = 0$$

$$(1.2) \quad v(0) = v'(b) = 0, \quad 0 < b,$$

where

(i) $f(x, v) = vg(x, v)$ and $g(x, v)$ is positive and of class C^1 in (x, v) on $R = \{(x, v) : x > 0, |v| < \infty\}$,

(ii) the following monotonicity conditions hold on R ,

$$(1.3) \quad f_v(x, v) > 0$$

$$(1.4) \quad vg_v(x, v) < 0, \quad v \neq 0,$$

(iii) for some $\delta > 0$, $xf_v(x, 0) \leq M < \infty$, provided $0 < x < \delta$, and finally either

(iv) $g(x, v)$ is even in v for fixed x , i.e.

$$g(x, v) = g(x, -v),$$

or at least

(iv') for each x , $0 < x < \infty$, $\lim_{v \rightarrow +\infty} g(x, v) = \lim_{v \rightarrow -\infty} g(x, v)$, (note that the existence of these limits is guaranteed by (1.4) and the fact that $g(x, v)$ is positive).

The equation (1.1) is allowed to have a singularity at the origin (see (i) and (iii) above) in order that the results below should contain certain results of Kolodner [1].

When $g(x, v)$ is of class C^1 on the closure of R , then what is known concerning the existence of solutions of the problem (1.1)–(1.2) can be summarized as follows. Set

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