

The Behavior of Increasing Stable Processes For Both Small and Large Times

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1. Introduction. In this paper we shall be concerned with the sample functions of increasing stable processes, $X_{\alpha\beta}$, having no deterministic linear component. If h is a fixed function, it is well-known ([1], page 57) that each of the following four events has probability zero or one:

$$\begin{aligned} &\{w : X_{\alpha\beta}(t, w) > h(t) \text{ infinitely often as } t \rightarrow 0\}, \\ &\{w : X_{\alpha\beta}(t, w) > h(t) \text{ infinitely often as } t \rightarrow \infty\}, \\ &\{w : X_{\alpha\beta}(t, w) \leq h(t) \text{ infinitely often as } t \rightarrow 0\}, \\ &\{w : X_{\alpha\beta}(t, w) \leq h(t) \text{ infinitely often as } t \rightarrow \infty\}. \end{aligned}$$

Khinchin ([5]) has given us integral tests to apply to a wide class of h 's in order to determine whether one, the other, or both of the first two events have probability zero or one. (Khinchin's results are not restricted to the case at hand.) The purpose of this paper is to give similar tests for the last two events. These tests are contained in theorems 3 and 4 in Section 3.

In Section 2 we restate some well-known facts and introduce some notation. In Section 4 we apply the results of Section 3 to certain specific functions h . In Section 5 we make some observations concerning generalizations of theorems 1 through 4.

2. Notation. For each $\alpha \in (0, 1)$ and $\beta \in (0, \infty)$ we shall consider the increasing stable process $X_{\alpha\beta}$; the correspondence between indices and stable processes to be described in what follows. $X_{\alpha\beta}$ is a non-negative real-valued function defined on $[0, \infty) \times \Omega$, where Ω is a probability space. For each $w \in \Omega$, $X_{\alpha\beta}(0, w) = 0$ and $X_{\alpha\beta}(\cdot, w)$ is a non-decreasing right continuous function. Moreover, we only consider those processes with no fixed linear component; $X_{\alpha\beta}(t, w)$ is equal to the sum of the jumps of $X_{\alpha\beta}(\cdot, w)$ up through time t .

The first part of Theorem 3 is essentially taken from the author's PhD thesis completed at MIT under the direction of Henry P. McKean, Jr.

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