

# *Construction of a Markov Process from Hitting Probabilities*

FRANK KNIGHT\* & STEVEN OREY\*

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Ever since the work of Hunt [2] hitting probabilities have played a prominent part in the study of Markov processes. In [1] it was shown that for a wide class of processes the hitting probabilities characterize a Markov process with stationary transition probabilities up to a change of time scale. The question naturally arises when a class of measures  $H_A(x, \cdot)$ , with  $x$  ranging over points of some space  $K$  and  $A$  over subsets of  $K$  is the class of hitting distributions for some Markov process with stationary transition probabilities and state space  $K$ .

When considering a Markov process  $X(t)$  and a set  $A$  of the state space let

$$T_A = \inf_{t \geq 0} \{t : X(t) \in A\}$$

be the first entrance time into  $A$ . Note that we depart from the customary definition by taking the infimum over all non-negative  $t$  and not just the positive  $t$ . The hitting probability measure  $H_A(x, \cdot)$  is determined by

$$H_A(x, B) = P[X(T_A) \in B \mid X(0) = x].$$

The processes which we shall deal with all have the property that they are absorbed at a point  $\Delta$  after a finite time with probability 1. The time till reaching  $\Delta$  is the *lifetime* of the process.

We shall consider a compact metric space with a distinguished point  $\Delta$ . We shall assume the existence of measures  $H_A(x, \cdot)$  and a function  $g(x)$  satisfying certain postulates. The purpose of the paper is to show that there exists a Markov process with stationary transition probabilities having the  $H_A(x, \cdot)$  as hitting probabilities and such that  $g(x)$  is the expected lifetime of the process when started at  $x$ . A precise statement, embodying also a uniqueness assertion will be found in the Theorem terminating the paper.

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