

The Cauchy Problem for Degenerate Parabolic Equations

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1. Let points of euclidean E^n be denoted by (s, x) , where s is a scalar and $x = (x_1, \dots, x_n)$. It is assumed that $0 \leq s \leq T$ where T is a positive number which is fixed throughout the paper. Let L be a linear partial differential operator of the form

$$L(u) = u_s + \sum_{i,j=1}^n a_{ij}(s, x) u_{x_i x_j} .$$

The symmetric matrix $a(s, x) = (a_{ij}(s, x))$ is nonnegative definite. If it is everywhere positive definite, then L is a parabolic operator. When some of the characteristic values of $a(s, x)$ are 0 we call L a degenerate parabolic operator.

In this paper we shall consider quasi-linear equations of the form

$$(1.1) \quad L(u) + F(s, x, u, u_x) = 0,$$

together with the terminal data

$$(1.2) \quad u(T, x) = U(x).$$

By Cauchy problem we mean that of solving (1.1) in the strip $0 \leq s \leq T$, $x \in E^n$ with the data (1.2). The gradient in the space variables x is denoted by

$$u_x = (u_{x_1}, \dots, u_{x_n}).$$

If L is degenerate the Cauchy problem need not have any solution in the classical sense. However, instead of (1.1)–(1.2) we can consider approximate Cauchy problems depending on a parameter ϵ in which L is replaced by a uniformly parabolic operator L^ϵ . For each $\epsilon > 0$ the approximate problem has a solution $u^\epsilon(s, x)$ and we show that if conditions (A_1) , (A_2) , (H_2) are satisfied by the functions $a(s, x)$, $F(s, x, u, p)$, $U(x)$, then u^ϵ tends to a limit u as $\epsilon \rightarrow 0^+$. Under more restrictive assumptions we show that the limit u is a generalized solution

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