

# *An Analytic Triangulation of an Arbitrary Real Analytic Variety*

JACK E. GRAVER

*Communicated by A. H. WALLACE*

## **Introduction.**

**Definition 1.** By a local analytic variety, we will mean a subset  $V$  of an open set  $U$  in  $E^n$  which is given as the set of common zeros of a finite number of real power series convergent in  $U$ . A subset  $V$  of  $E^n$  will be called an analytic variety, if for each  $p \in V$ , there exists an open neighborhood  $U_p$  of  $p$  such that  $V_p = V \cap U_p$  is a local analytic variety.

**Definition 2.** By an analytic triangulation of a set  $S \subset E^n$ , we shall mean a homeomorphism  $\tau : K \rightarrow S$  (where  $K$  is a simplicial complex) such that  $\tau$  restricted to the interior of any simplex of  $K$  is analytic and  $\tau^{-1}$  restricted to the image of the interior of any simplex of  $K$  is analytic.

This paper proves the existence of an analytic triangulation of an open set  $O \subset E^n$  such that each of a finite number of analytic varieties closed in  $O$  appear as the image of a subcomplex. The construction of this analytic triangulation is carried out by means of an intermediate cell decomposition. We first decompose  $O$  into objects we choose to call analytic cells and then refine this decomposition with an analytic triangulation. Basically an analytic  $r$ -cell is an  $r$ -cell in the usual sense with the additional properties that it lies entirely within an  $r$ -dimensional local analytic variety and that its boundary on this variety is an analytic cell complex of lower dimension.

In part I, we give the precise definitions for analytic cell and analytic cell complex, and we develop the important properties of these objects. We end this part by proving that each analytic cell complex admits an analytic triangulation.

Part II is devoted entirely to proving the existence of an analytic cell decomposition of an open set  $O \subset E^n$  such that each of a finite number of analytic varieties closed in  $O$  appears as a subcomplex.

## **I. ANALYTIC CELLS AND THEIR PROPERTIES.**

**Definition 3.** An analytic 0-cell is a point in a Euclidean space. An analytic 0-cell is a face of itself and the point is its interior. The set of analytic cells is