

Addendum to “Deformations of Linear Connections and Riemannian Manifolds”

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1. We indicate here a generalisation of the results of our earlier paper (specifically Theorems 8.1, 9.1 and 9.2 in [1]), which is useful for some applications. The generalisation consists in varying also a tensorfield along with the connexion.

Let $(W, B, \pi : W \rightarrow B)$ be a locally trivial differentiable (of class C^∞) fibre space with total space W , base B and typical fibre M_0 . For $b \in B$, let w_b be a linear connection on $\pi^{-1}(b)$. Let $\mathbf{T} = \{T_b\}_{b \in B}$ be a C^∞ “tensorfield along the fibres” (see §2C, [1]).

Definition. The collection $(w_b)_{b \in B}$ is a \mathbf{T} -deformation of a connection w_0 on M_0 if

(i) there is a connection-preserving diffeomorphism of (M_0, w_0) onto $(\pi^{-1}(b), w_b)$ for some $b \in B$ and

(ii) there exists an open covering $(U_i)_{i \in I}$ of W , diffeomorphisms (into) $\varphi_i : W_i \rightarrow M_0 \times \pi(W_i)$, linear connections w_i on M_0 and tensorfields T_i on M_0 such that the diagram

$$\begin{array}{ccc} W_i & \xrightarrow{\varphi_i} & M_0 \times \pi(W_i) \\ \pi \searrow & & \swarrow p_2 \\ & \pi(W_i) & \end{array}$$

is commutative, where, p_2 is the natural projection; further if $p_1 : M_0 \times \pi(W_i) \rightarrow M_0$ is the other projection, $p_1 \circ \varphi_i$ is a connection-preserving diffeomorphism of $\pi^{-1}(b) \cap W_i$ into M_0 and also takes the tensorfield T_b into T_i .

Definition 2. A \mathbf{T} -deformation family (w_b, T_b) is locally trivial at $b_0 \in B$, if there is an open set $U \ni b_0$, a connection w_0 on M_0 , a tensor-field T_0 on M_0 and a homomorphism Φ (of fibre spaces) of $\pi^{-1}(U)$ on $M_0 \times U$ which on each fibre is connection-preserving and, for each $b \in B$, maps T_b into T_0 .

Imitating the procedure of [1], we obtain an “infinitesimal deformation map” δ and, for each $b \in B$, a “punctual deformation map” $\delta_b : V_b \rightarrow H^1(\pi^{-1}(b), \Theta_b)$