

*Absolute Parallelism in a Conformally Flat V_4^**

V. HLAVATÝ

1. Problem to be solved. It is a common knowledge that the four dimensional Riemann space V_4 is conformally flat if and only if its curvature tensor $K_{\omega\mu\lambda\nu}$ is of the form

$$(1.1) \quad K_{\omega\mu\lambda\nu} = -g_{\lambda[\omega}K_{\mu]\nu} + g_{\nu[\omega}K_{\mu]\lambda} + K/3g_{\lambda[\omega}g_{\mu]\nu} \neq 0.$$

Here

$$(1.2) \quad \begin{aligned} K_{\mu\lambda} &\stackrel{\text{def}}{=} K_{\omega\mu\lambda\nu}g^{\omega\nu}, \\ K &\stackrel{\text{def}}{=} K_{\mu\lambda}g^{\mu\lambda}. \end{aligned}$$

The purpose of this short paper is to find necessary and sufficient conditions that a conformally flat V_4 with metric of the signature $+2$ admits a field of absolute parallel vectors v_λ

$$(1.3)a \quad \nabla_\mu v_\lambda = v_\lambda \partial_\mu \ln \varphi, \quad \varphi \neq 0 \quad \text{arbitr.}$$

(∇_μ is the symbol of covariant derivative with respect to the Christoffel symbols of the metric tensor.) It will turn out that these conditions may be fully described by the (Weierstrass) characteristic of $K_{\mu\lambda}$ and by the condition imposed on K .

Remark (1.1). We limit ourselves to the case $n = 4$ because it is of particular interest to the Theory of Relativity. The results obtained here may be easily generalized for an arbitrary n .

Remark (1.2). An easy inspection shows that (1.3)a is equivalent to

$$(1.4) \quad \nabla_\mu u_\lambda = 0,$$

for

$$(1.3)b \quad v_\lambda = \varphi u_\lambda.$$

Therefore, we can limit ourselves to the case (1.4). The equation (1.4) shows

* This paper was prepared with the support of the Aerospace Research Laboratories of the Office of Aerospace Research U. S. A.