

# *An Asymptotic Method for Solving Perturbed Wiener-Hopf Problems\**

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**1. Introduction.** Perturbed Wiener-Hopf problems occur in many branches of applied mathematics. As a typical example, we may consider the perturbed inhomogeneous Wiener-Hopf integral equation

$$(1.1) \quad \int_0^{\infty} k^*(t - t_0)f^*(t_0) dt_0 = h(t), \quad t > 0,$$

in which  $h$  and  $k^*$  are known functions and  $f^*(t)$  is unknown. We suppose that  $k^*$  is of the form

$$(1.2) \quad k^*(t) = k(t) + \epsilon m(t),$$

$\epsilon$  being some positive parameter, and that we can find the solution  $f(t)$  of the "unperturbed" integral equation

$$(1.3) \quad \int_0^{\infty} k(t - t_0)f(t_0) dt_0 = h(t), \quad t > 0.$$

We then seek an asymptotic representation, valid for small  $\epsilon$ , of  $f^*(t)$  in terms of  $f(t)$ .

Very often, such an asymptotic solution can be found by a straightforward iterative procedure (*cf.* [1], p. 163). Difficulties arise, however, whenever the perturbing term  $m(t)$  is more singular (or less smooth) at the origin than the unperturbed kernel  $k(t)$ . It is with such cases that we deal in this paper.

In order to describe the precise situation we shall discuss, we introduce Laplace transforms. We extend (1.1) to the negative  $t$ -axis in the usual way by introduc-

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