

Some Further Results on Central Limit Theorems for Non-linear Functions of a Normal Stationary Process*

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Introduction. In [1] the following theorem was proved (see Theorem 2 and Theorem 3 in [1]):

Theorem 0. If X_t , $t = 0, \pm 1, \pm 2, \dots$ is a real normal stationary process with mean $EX_t = 0$ for all t and covariances

$$r_k = EX_t X_{t+k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ik\lambda} f(\lambda) d\lambda, \quad k = 0, \pm 1, \pm 2, \dots,$$

where $f \in L^2(-\pi, \pi)$ and

$$(1) \quad \lim_{N \rightarrow \infty} \frac{1}{2\pi N} \int_{-\pi}^{\pi} \frac{\sin^2 \frac{N}{2} \lambda}{\sin^2 \frac{1}{2} \lambda} f(\lambda) d\lambda$$

exists and is finite, then the following n random variables

$$Y_{N, \alpha} = N^{-\frac{1}{2}} \sum_{j=1}^N \left[\prod_{i=1}^{m_\alpha} X_{t+k_j^{(\alpha)}} - E \left(\prod_{i=1}^{m_\alpha} X_{t+k_j^{(\alpha)}} \right) \right],$$

$\alpha = 1, 2, \dots, n$, where m_α are positive integers and $k_j^{(\alpha)}$ are integers, are asymptotically jointly normally distributed with mean zero and some finite covariances. In particular, if all the m_α are even integers, the condition (1) is not needed.

Hence if

$$Y_t = \sum_{m=1}^M \sum_{\substack{k_1, \dots, k_m \\ k_1 = -K, \dots, m \\ k_1 - 1 \leq k_1}}^K a_{k_1, \dots, k_m} \left[\prod_{j=1}^m X_{t+k_j} - E \left(\prod_{j=1}^m X_{t+k_j} \right) \right],$$

where a_{k_1, \dots, k_m} are any constants and M and K are any positive integers, then

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