

# *Pointwise and Norm Convergence of Distributions<sup>†</sup>*

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**Abstract.** In this paper we derive some results concerning the relation between ordinary pointwise convergence, convergence in the sense of Schwartz distributions (weak convergence), and norm convergence of locally integrable functions by exploiting the notion of “analytic continuation” for distributions which was recently investigated by H. J. Bremermann and L. Durand III (Jour. Math. Phys. 1961). One of our results is that weak, normed, and almost everywhere pointwise convergence are the same for every dominated sequence provided a certain local equicontinuity in the mean condition is satisfied. A partial converse to this result, showing that norm convergence must imply the “local equicontinuity” condition and dominance as well as pointwise convergence for some subsequence, is also established.

**Section I: Introduction.** L. Schwartz in [1] introduced the following extension of the class of locally integrable complex valued functions on the real line (space  $L$ ). Let  $D$  be the linear space of all  $C^\infty$  functions having compact support on the real axis with convergence in  $D$  being uniform convergence in each derivative for sequences having their supports uniformly contained in some bounded set.  $D'$ , the dual of  $D$ , is the space of Schwartz distributions.  $L$  is imbedded in  $D'$  since every  $f$  in  $L$  defines an element of  $D'$  via the relation

$$(1.1) \quad \langle f, \phi \rangle = \int_{-\infty}^{\infty} f\phi \, dx.$$

The functional (1.1) is uniquely defined for all  $\phi$  in  $D$  by any function, which is almost everywhere equal to  $f$ .  $D'$  has the important property of being closed with regard to weak convergence: if a sequence  $\Theta_n$  in  $D'$  is such that  $\Theta_n(\phi)$  has

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