

Weierstrass' Characteristic and Covariant Constant Vector Fields

*I. Preferred Vector Fields**

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1. Problems to be solved. In the general theory of relativity one assumes very often that the metric form is

$$(1.1) \quad ds^2 = \sum_{\xi, \eta}^{I, II, III} g_{\xi\eta} dx^\xi dx^\eta - c^2(dx^{IV})^2,$$

where $g_{\xi\eta}$ is of rank 3 and positive definite. Thanks to this special form one obtains often satisfactory and nice results provided that *only such transformations of coordinates are admitted which preserve the form on the right hand side of (1.1)*. However, in many cases this (italicized) requirement applied without further qualification is against the spirit of the theory of relativity, where all coordinates have to be dealt with as equivalent. On the other hand, there are cases where the form (1.1) is imposed on our space V_4 by the structure of the metric tensor $g_{\lambda\mu}$ expressed in an arbitrary coordinate system. In such cases the form (1.1) displays some properties of V_4 which are *independent of the choice of the coordinate system*. In order to avoid any misunderstanding we shall stress this requirement in the following:

Definition (1.1). *The right hand side of (1.1) will be termed a natural form if the following conditions are satisfied*

1) *The geometrical content of the right hand side of (1.1) can be described without the use of any coordinate system.*

2) *This description is a concomitant of $g_{\lambda\mu}$ and its derivatives only, i.e. does not bring into V_4 any extraneous element given besides $g_{\lambda\mu}$.*†

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† The geometrical content of the metric form (1.1) is that $x^\xi = \text{const}$ ($\xi = I, II, III$) is a normal geodesic congruence C of curves. If C is not a concomitant of $g_{\lambda\mu}$ (and its derivatives) only, then it represents an extraneous element in V_4 given *besides* $g_{\lambda\mu}$. In this case the condition 2) is not satisfied.