

Torsion in Link Modules

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1. Introduction. If H is a multiplicative group, we denote its integer group ring by $Z(H)$. In [1] it is shown that every exact sequence $1 \rightarrow K \rightarrow G \xrightarrow{\epsilon} H \rightarrow 1$ of groups determines an exact sequence

$$(1) \quad 0 \rightarrow B \rightarrow A \rightarrow I \rightarrow 0$$

of left $Z(H)$ -modules. In this sequence I is the augmentation ideal of H , *i.e.*, the kernel of the augmentation mapping, or trivializer, $\epsilon: Z(H) \rightarrow Z$. The module B is the commutator quotient group of K written additively with the action of H defined by conjugation in G , *e.g.*, see [4, p. 108]. If G has a group presentation $G = \langle x_j : r_i \rangle$, then the Jacobian matrix $\|\theta_j(\partial r_i / \partial x_j)\|$ is a relation matrix for the module A .

Let $L \subset S^3$ be a tame link with $m \geq 1$ components. We study the module sequence (1) obtained by taking $G = \pi_1(S^3 - L)$, and $H = G/G' = H_1(S^3 - L)$, and $K = G'$. The group H is free abelian with m generators. If $m = 1$, *i.e.*, if L is a knot, then the augmentation ideal I is principal. As a result, it is a free module, and so (1) splits giving $A = B \oplus Z(H)$. Furthermore, for knots, B is the torsion submodule of A and is annihilated by the first elementary ideal of A , and therefore also by the Alexander polynomial of A , see [2]. In this paper analogous results are obtained for $m \geq 2$, *i.e.*, for links as opposed to knots. We shall prove the following four theorems.

(1.1) *The first elementary ideal E_1 of A annihilates B . If $E_1 \neq 0$, then B is the torsion submodule of A .*

The second assertion in (1.1) is a straightforward corollary of the first because if H is free abelian, the ring $Z(H)$ has no divisors of zero.

(1.2) *If the product $(t_1 - 1) \cdots (t_m - 1)$ does not divide the Alexander polynomial Δ_1 of A , then B is annihilated by Δ_1 .*

Suppose that no two components of L have linking number zero. It follows easily by induction on m and the known properties of Δ_1 , see [6, p. 57], that the product $(t_1 - 1) \cdots (t_m - 1)$ does not divide Δ_1 .

(1.3) *The sequence (1) never splits if $m \geq 3$.*

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