

Spinor Representations of Infinite Orthogonal Groups

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Communicated by V. BARGMANN

0. Introduction. A real Hilbert space H may be imbedded in a complex algebra by requiring that $xy + yx = 2(x, y)1$. This is the Clifford algebra of H . It will be denoted by $C_0(H)$. It extends in a unique way to a C^* -algebra $C(H)$ in which all the elements of H are self-adjoint. This is shown in [5] section 1 and is discussed briefly in section 3 below.

The Clifford algebra arises in the theory of mechanical systems with finite or infinitely many degrees of freedom which satisfy Fermi-Dirac statistics. (See for example the discussion in the introduction of [5].) Quantizing such a system amounts to prescribing a real-linear map from a complex Hilbert space H to self-adjoint operators which satisfy the anti-commutation relations. The complex Hilbert space H determines a real Hilbert space with inner product $\operatorname{Re}(x, y)$. We denote the Clifford algebra over this real space again by $C_0(H)$. A quantization then amounts to a $*$ -representation of $C_0(H)$. The representation occurring most often in physics is the Fock quantization. This is unitarily equivalent to the holomorphic spinor representation given in [5] and described in section 1 below.

In section 2 we take a complex Hilbert space H and the holomorphic spinor representation F of $C_0(H)$. If T is a transformation on H which is orthogonal with regard to the underlying real Hilbert space structure then the map $x \rightarrow Tx$ on H extends uniquely to a $*$ -automorphism \mathfrak{J} of $C_0(H)$. We determine those T for which there exists a unitary operator U on the representation space such that $F(X^{\mathfrak{J}}) = U^{-1}F(X)U$ for all X in $C_0(H)$. A special case has been treated by Segal in [3] example 3.4, p. 40. Our procedure is to adapt the methods of [3] and [4] to reduce the general case to what is essentially this special one. Since the holomorphic spinor representation is irreducible, the unitary operator U above is determined, when it exists, up to a scalar factor of absolute value 1.

¹ Supported in part by the National Science Foundation grant NSF-GP780.