

*Statistical Properties of the Stress and Strain Fields in a Medium with Small Random Variations in Elastic Coefficients**

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Abstract. In this paper we consider a statistically homogeneous medium with small random variations in elastic properties. We derive the equations satisfied by the two point displacement field correlation tensor and the cross correlation tensors between the strain field and the elastic coefficients $\lambda(x)$ and $\mu(x)$ for a medium subjected to an average linear displacement. The problem of a statistically homogeneous and isotropic elastic medium under hydrostatic pressure and subject to the assumption of small perturbations is then treated in detail. Here, we compute the effective bulk modulus and show that for two phase materials it is the same as that derived by Hashin and Shtrikman (1963) if we take a limiting form of their bounds. The stress and strain field correlation tensors are also calculated, and the order of magnitude of the mean square values of stress and strain computed from them is shown to be in agreement with the assumptions which have been made.

I. Introduction. In a recent paper Beran and Molyneux (1963) considered the statistical properties of an electric field in a medium with small random variations in permittivity. The techniques developed in this paper are here extended to treat the statistical properties of the stress and strain fields in a medium with small random variations in elastic constants subjected to an average linear displacement. The basic approach is similar to that used in deriving the statistical equations governing turbulent diffusion phenomena. Here, however, it is perhaps more appropriate to use perturbation theory and thus obtain a series of closed equations.

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