

On the Construction of Some Integral Operators for Generalized Axially Symmetric Harmonics and Stream Functions

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Summary. Integral transformations are developed for the Stokes Beltrami equations from essentially one identity known for axially symmetric harmonics and stream functions in several space dimensions. It is shown further how pairs of solutions can be transformed from a given dimension to a higher or lower one. Application is also made to the repeated operator equations, and the fundamental singularities for the axisymmetric flow of an incompressible irrotational fluid are generated from corresponding complex potentials for plane flows.

1. Introduction. The following work is partly a generalization of [1] and related to [2] where integral transformation formulae were given for the Stokes Beltrami equations in various orthogonal curvilinear coordinate systems and which transform solutions from a given dimension to a dimension higher by one and vice versa. It is shown here how these transformations can be developed by means of two known identities due to Weinstein [3], [4], and which form the basis of many results arising from generalized axially symmetric potential theory. The first identity is associated with a differential operator where the independent variable is proportional to the area generated by rotating the cylindrical radius vector about the axis of symmetry and its effect is to increase the dimension of a harmonic by two, and reduce that of a stream function by two. The second identity is the correspondence principle which relates adjoint solutions of the axially symmetric Laplace equation in a space of several dimensions. The representations for velocity potential and stream function are combined to generate pairs of solutions in a given dimension in terms of pairs in a different dimension. In particular it is shown how a pair of solutions in a given dimension can be constructed from functions satisfying the Cauchy Riemann equations.