

On Filtered Lie Algebras and Geometric Structures II

SHOSHICHI KOBAYASHI AND
TADASHI NAGANO*

Communicated by S. S. CHERN

1. Introduction. In our previous papers [4], [5], we obtained a structure theorem on certain filtered Lie algebras and promised its applications to G -structures and transformation groups acting on symmetric spaces. In the present paper we give its applications to transformation groups.

The problem considered here may be stated as follows: Given an irreducible Riemannian symmetric space M , does there exist a Lie transformation group of M which contains the group of isometries of M ? Some typical examples are: (i) the projective transformation group acting on a projective space, (ii) the conformal transformation group acting on a Möbius space, (iii) the holomorphic transformation group acting on a compact Hermitian symmetric space. In these examples the spaces are all compact. On the other hand, for a symmetric bounded domain (in fact, for any bounded domain), its holomorphic transformation group consists of isometries (with respect to the Bergman metric). In [9] Nagano obtained the classification of connected Lie transformation groups of compact symmetric spaces M which contain the largest connected groups of isometries of M . In this paper we shall give a similar classification for irreducible symmetric spaces. In particular, it will be shown that an irreducible noncompact symmetric space does not admit, roughly speaking, a Lie transformation group larger than the group of isometries (see Corollaries 3 and 4 below for exact statements).

At this point we may also indicate the bearing this paper has on the classification problem of G -structures. Let M be an irreducible compact Riemannian symmetric space. Then two isometries of M coincide if they have the same differential at a single point of M . In other words, an isometry is uniquely determined by its first partial derivatives at one point. Assume now that M admits a Lie transformation group L larger than the group of isometries (see

* Both authors are partially supported by NSF Grant GP 812.